

XVIII. *On the Relation of Form and Dimensions to Weight of Material in the Construction of Iron-clad Ships.* By E. J. REED, Chief Constructor of the Navy. Communicated by Professor G. G. STOKES, Sec. R.S.

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THE object of the present paper is to show that the proportion of length to breadth in a ship, and the form of her water-lines, should be made in a very great degree dependent upon the weight of the material of which her hull is to be constructed—that an armour-plated ship, for example, should be made of very different proportions and form from those of a ship without armour, and that as the extent and thickness of the armour to be carried by a ship are increased the proportion of length to breadth should be diminished, and the water-lines increased in fulness.

It is highly desirable that this subject should receive the attention of men of science, not only because it bears most directly upon both the cost and the efficiency of future iron-clad fleets, but also because it opens up a theoretical question which has hitherto, I believe, received absolutely no consideration from scientific writers upon the forms and resistances of ships, viz. the manner in which the weight of the material composing the hull should influence the form.

Prior to the design of the ‘Bellerophon,’ the forms of ships were determined in complete disregard of this consideration; and even the most recent works upon the subject incite the naval architect to aim always at approaching the form of least resistance. The following investigations will show, however, that the adoption of a form of least resistance, or of small comparative resistance, may, in fact, lead to a lavish outlay upon our ships, and to a great sacrifice of efficiency; while, on the other hand, the adoption of a form of greater resistance would contribute in certain classes of ships to great economy and to superior efficiency.

In order to indicate clearly, but approximately only, the purpose in view, I will first assume that figs. 1 & 2 roughly represent a long and a shorter ship respectively, both being prismatic in a vertical sense. The length of fig. 1 is seven times its breadth; that of fig. 2 five times its breadth, the middle portion of the latter being parallel for two-fifths of its length. I will further assume that the ship (fig. 1) will give a constant of 600, and fig. 2 a constant of 500 in the Admiralty formula

$$\frac{\text{speed}^3 \times \text{midship section}}{\text{indicated horse-power}} = \text{constant.}$$

The draught of water is in each case 25 feet, and the total depth 50 feet.

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I will take it for granted that the form fig. 1 has been found satisfactory for a ship of such scantlings that we may consider her to be built of iron of a uniform thickness

Fig. 1.

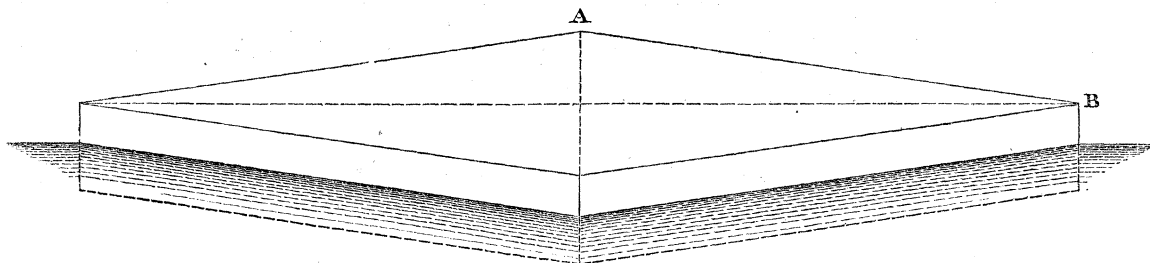
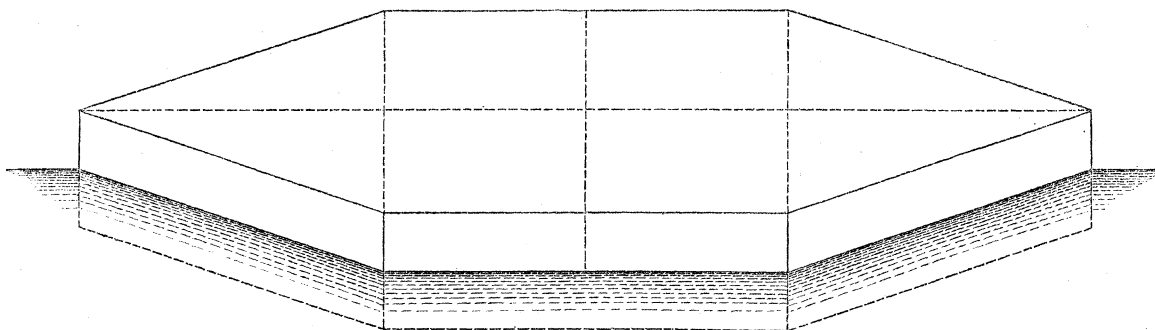


Fig. 2.



of 6 inches, the top and bottom being weightless, and I will take fourteen knots as the speed at which the constant 600 was obtained.

Now, let it be required to design a ship of equal speed, draught of water, and depth, but of such increased scantlings (whether of hull proper or of armour) that the weight shall be equivalent to a uniform thickness of 12-inch iron, the top and bottom being weightless as before. First, we will give to this new ship the proportions and form of fig. 1; secondly, we will give her the form and proportions of fig. 2.

In each case the ship shall be engined with engines developing seven times their nominal horse-power, and weighing (with boilers, water, &c.) 1 ton per nominal H.P.

In order that both ships may steam the same distance at the same speed, we will give them a coal-supply equal in each case to the weight of the engines; but as the smaller ship will require less men and provisions, and a less weight of other stores, we will require the larger ship to carry 2000 tons, and the smaller but 1500 tons of such additional weights.

Taking, first, the long ship fig. 1, and assuming $2x$ to be the breadth, we have

$$\begin{aligned} \text{I.H.P.} &= \frac{14^3 \times 50x}{600} \\ &= 228x. \end{aligned}$$

Therefore weight of engines in tons $= \frac{228x}{7} = 32.5x$ tons.

$$\begin{aligned} \text{Weight of hull} &= 4 \cdot \overline{AB} \times 50 \times 12 \times \frac{40}{2 \cdot 240} \text{ tons} \\ &= 4\sqrt{50}x \times \frac{75}{7} \\ &= 214\sqrt{2}x. \end{aligned}$$

Therefore, weight of hull, engines, and coal

$$\begin{aligned} &= 214\sqrt{2}x + 2 \times 32 \cdot 5x \\ &= (65 + 214\sqrt{2})x, \end{aligned}$$

and displacement, or total weight,

$$= 2000 + (65 + 214\sqrt{2})x.$$

But displacement is also equal to

$$\begin{aligned} &\frac{14x \times 2x}{2} \times 25 \times \frac{1}{35} \\ &= 10x^2; \end{aligned}$$

$$\therefore 10x^2 = 2000 + (65 + 214\sqrt{2})x,$$

whence

$$x = 41 \cdot 5.$$

The following, therefore, are the dimensions &c. of the new ship of the fig. 1 type, viz.—

Length	= 14x	=	581 feet.
Breadth	= 2x	=	83 „
Nominal horse-power	= 32·5x	=	1350 H.P.
Indicated horse-power	= 228x	=	9450 „
Weight of hull	= 214√2x	=	12570 tons.
Weight of engines	= 32·5x	=	1350 „
Weight of coals	= 32·5x	=	1350 „
Weight carried		=	<u>2000</u> „
Total displacement			17270 tons.

Taking next the shorter ship of fig. 2 type, and assuming 2x to be the breadth, as before, and therefore 10x the length, and 4x the length of the parallel portion, we have

$$\begin{aligned} \text{I.H.P.} &= \frac{14^3 \times 50x}{500} \\ &= 274x. \end{aligned}$$

Pursuing the same process as before, and equating again the two values of the displacement, we shall have

$$10x^2 = 1500 + 299x$$

and

$$x = 34 \cdot 28.$$

The following, therefore, are the dimensions &c. of the new ship of the shorter type, viz.—

Length	= $10x = 342$	feet.
Breadth	= $2x = 68\frac{1}{2}$,,
Nominal horse-power	$= \frac{274x}{7} = 1337$	H.P.
Indicated horse-power	$= 274x = 9359$,,
Weight of hull . . .	$= 221x = 7576$	tons.
Weight of engines . .	$= 39x = 1337$,,
Weight of coals . . .	$= 39x = 1337$,,
Weight carried	$= 1500$,,
Total displacement . .	$= 11750$,,

It will therefore be seen that, by adopting in the new design the proportions and form of fig. 2, a ship of the required scantlings and speed will be obtained on a length of 342 feet, and a breadth of $68\frac{1}{2}$ feet; whereas if the proportions and form of fig. 1 are adopted, the ship, although of the same scantlings and speed only, will require to be 581 feet long and 83 feet broad, the steam-power being in both cases as nearly as possible the same.

It was by considerations of the above nature, worked out more fully and corrected by the results of the trials of many large ships of war, that in designing the 'Bellerophon' I was led to depart so considerably from the proportions and form of the 'Warrior,' 'Minotaur,' and other long iron-clads.

We are now in possession of the actual performances of these ships, and it is upon these as a basis that I propose to continue the present investigation.

From the drawings of the 'Minotaur' and of the 'Bellerophon,' and from calculations based upon them, also from the official reports of the measured mile trials made with those ships rigged for sea, the following particulars are obtained:—

	'Minotaur.'	'Bellerophon.'
Mean displacement	9492 tons.	6808 tons.
Mean midship section	1235 sq. ft.	1127 sq. ft.
Mean of mean draughts of water	24' 11"	22' 10 $\frac{1}{2}$ "
Weight of hull	5092 tons.	3477 tons.
Weight of backing to armour	335 ,,	165 ,,
Weight of armour	1776 ,,	1218 ,,
Weight of engines, coals, and stores	2289 ,,	1948 ,,

It follows from the above that the areas of the mean horizontal sections, which must be equal to the mean displacement in cubic feet divided by the mean draught of water, are

$\frac{9492 \times 35}{24\frac{11}{12}} = 13333 \text{ sq. ft.}$	$\frac{6808 \times 35}{22\frac{10}{8}} = 10407 \text{ sq. ft.}$
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The draughts of water which are found to correspond with water-line areas of these sections are,

'Minotaur'	9.6 feet.
'Bellerophon'	8.3 feet.

Supposing the mean sections at these depths to be parallel to the construction water-lines, we find from the drawings that their dimensions are—

	‘Minotaur.’	‘Bellerophon.’
Half breadth extreme	26·8 ft.	26 ft.
Length measured along the side	394 ft.	305·5 ft.

We also find from the drawings that the mean lengths of the horizontal sections of the armoured sides are—

‘Minotaur,’ 420 ft.	‘Bellerophon,’ 320 ft.
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Let us now suppose that the weight of the armour and backing is uniformly distributed in each case over vertical prismatic sides of the dimensions of the armoured sides, and that the weight of the hull is similarly distributed over vertical prismatic sides of the dimensions below water of the mean horizontal section, and above water of the armoured side, observing that the total depth of the armour and backing on the ‘Minotaur’ is 22 feet, and on the ‘Bellerophon’ 20·3 feet, and that the ‘Minotaur’s’ armour descends $5\frac{3}{4}$ feet below water, and the ‘Bellerophon’s’ 6 feet.

Then we shall have

	‘Minotaur.’	Tons.	‘Bellerophon.’	Tons.
Weight per sq. ft. of armour and backing	$\frac{2111}{2 \times 420 \times 22}$	=·1142;	$\frac{1383}{2 \times 320 \times 20\cdot3}$	=·1064.
	‘Minotaur.’	Tons.	‘Bellerophon.’	Tons.
Weight of hull	$\frac{5092}{2 \times 394 \times 24\frac{1}{2} + 2 \times 420 \times 16\frac{1}{4}}$	=·153;	$\frac{3477}{2 \times 305\cdot5 \times 22\frac{7}{8} + 2 \times 320 \times 14\cdot3}$	=·1503.

In these two classes of ships, therefore, we have very similar conditions as respects weight per sq. ft. both of hull and of defensive material carried (armour and backing).

The ‘Minotaur’ has slightly the heavier weight per sq. ft. for both, but as the ‘Bellerophon’ has a large reserve of strength compared with ‘Minotaur,’ we may without error take the mean weights per sq. ft. both of hull and of armour and backing to be the same. We shall thus have in round numbers for both ships—

Weight per sq. ft. of hull	=·152 ton.
” ” armour and backing	=·11 ”

Here, then, we have two types of ship (the ‘Minotaur’ 400 feet long between perpendiculars and 59 feet $3\frac{1}{2}$ inches broad, and the ‘Bellerophon’ 300 feet long and 56 feet broad), both steaming on the measured mile at fourteen knots, and both having equal weights per sq. ft. of hull and of defensive armour and backing; and it has been ascertained that on their measured mile trials, which were conducted in precisely the same way, and with both ships fully rigged, they have given the following mean constants of performance calculated from the formula

$$C = \frac{\text{mid. sec.} \times \text{speed}^3}{\text{I.H.P.}}, \text{ viz.}$$

	‘Minotaur.’	‘Bellerophon.’
Mean constant.	572·9	516·5

The questions I now wish to consider are these: presuming it to be necessary to build another ship, which shall steam at the same speed, carry the same proportionate supply of coal to engine-power and proportionate quantities of stores, but shall have her armour and backing of double the weight of the armour and backing of the 'Bellerophon' and 'Minotaur,' then (1) what will be the size, engine-power, and cost of the new ship if built of the 'Minotaur' type, and having the same mean draught and depth of armour? and (2) what will be her size, engine-power, and cost if built of the 'Bellerophon' type, and having her mean draught and depth of armour?—this condition implying, of course, that the same constants of performance as before will be realized in each case. Looking to the great disproportion of size between the two types of ship, it is obvious that the smaller one will require a much less weight of stores, crew, &c. than the larger vessel; we will therefore take the weight to be carried by the latter, in addition to the weight of engines, boilers, and coals, as 1000 tons, and that to be carried by the former 700 tons. The engines in each case are to weigh 1 ton per nominal H.P., and to work up to seven times that power. The coal is to be equal to the weight of the engines in each case. Taking first the case of the new ship of the 'Minotaur' type, and calling the half breadth of her mean level section b , we shall have (since for similar curves the lengths are proportional to a dimension, and the areas to the square of a dimension)

$$\begin{aligned}
 \left. \begin{array}{l} \text{Length along the side of the} \\ \text{mean level section} \end{array} \right\} &= \frac{394}{26.8} \quad b = 14.7b. \\
 \text{Area of the mean level section} &= \frac{13333}{26.8^2} b^2 = 18.56b^2. \\
 \text{Length of armoured side} &= \frac{420}{26.8} \quad b = 15.67b. \\
 \text{Length between perpendiculars} &= \frac{400}{26.8} \quad b = 14.9b. \\
 \left. \begin{array}{l} \text{Area (approximate) of im-} \\ \text{mersed mid. sec.} \end{array} \right\} &= \frac{1235}{26.8} \quad b = 46.08b. \\
 \text{Area of armoured sides} &= 2 \times 15.67b \times 22 = 689.5b. \\
 \text{Area of surface for hull} &= 2 \left\{ 14.7b \times 24\frac{1}{2} + 15.67b \times 16\frac{1}{4} \right\} = 1241.8b. \\
 \text{Weight of armour and backing} &= 689.5b \times .22 \text{ tons} = 151.69b \text{ tons.} \\
 \text{Weight of hull} &= 1241.8b \times .152 \text{ tons} = 188.75b \quad . . . \quad (1)
 \end{aligned}$$

Also we have, for the new ship,

$$\text{I.H.P.} = \frac{14^3 \times \text{mid. sec.}}{572.9} = \frac{2744 \times 46.08b}{572.9} = 220.7b.$$

Therefore

$$\text{Nominal horse-power} = \frac{1}{7} \text{I.H.P.} = 31.53b,$$

and

$$\text{Weight of engines and coals} = 2 \text{ nom. H.P.} = 63.06b \text{ tons.}$$

Now,

$$\text{Displacement in tons} = \frac{\text{area of mean level sec.} \times \text{draught}}{35} = \frac{18 \cdot 56b^2 \times 24 \frac{1}{2}}{35} = 13 \cdot 2b^2.$$

And displacement is also equal to weight of hull + weight of armour and backing + weight of engines and coals + 1000 tons. Whence we have the equation

$$13 \cdot 2b^2 = 188 \cdot 75b + 151 \cdot 69b + 63 \cdot 06b + 1000; \dots \dots \dots (A)$$

$$\therefore 13 \cdot 2b^2 - 403 \cdot 5b = 1000,$$

and

$$b = 32 \cdot 87.$$

From this value of b we get the following dimensions and particulars of the new ship of the 'Minotaur' type with thick armour, viz.—

Length between perpendiculars	= $14 \cdot 9b$	= 489·7 feet.
Breadth extreme	= $\frac{489 \cdot 7}{400} \times 59 \frac{7}{24}$	= 72·5 „
Displacement	= $13 \cdot 2b^2$	= 14253 tons.
Indicated horse-power	= $220 \cdot 7b$	= 7254 H.P.
Nominal horse-power	= $\frac{220 \cdot 7}{7} b$	= 1036½ „
Weight of hull	= $188 \cdot 75b$	= 6194 tons.
Weight of armour and backing	= $151 \cdot 69b$	= 4986 „
Weight of engines and coals	= $63 \cdot 06b$	= 2073 „

Taking now the new ship of the 'Bellerophon' type, and calling the half breadth of her mean level section b' , we shall have, by a similar process to the preceding,

$$\left. \begin{array}{l} \text{Length along the side of the} \\ \text{mean level section} \end{array} \right\} = \frac{305 \cdot 5}{26} b' = 11 \cdot 75b'.$$

$$\text{Area of mean level section} = \frac{10407}{26^2} b'^2 = 15 \cdot 4b'^2.$$

$$\text{Length of armoured side} = \frac{320}{26} b' = 12 \cdot 3b'.$$

$$\text{Length between perpendiculars} = \frac{300}{26} b' = 11 \cdot 54b'.$$

$$\left. \begin{array}{l} \text{Area (approximate) of im-} \\ \text{mersed mid. sec.} \end{array} \right\} = \frac{1127}{26} b' = 43 \cdot 3b'.$$

$$\text{Area of armoured sides} = 2 \times 12 \cdot 3b' \times 20 \cdot 3 = 499 \cdot 38b'.$$

$$\text{Area of surface for hull} = 2 \{ 11 \cdot 75b' \times 22 \frac{7}{8} + 12 \cdot 3b' \times 14 \cdot 3 \} = 889 \cdot 54b'.$$

$$\text{Weight of armour and backing} = 499 \cdot 38b' \times \cdot 22 \text{ ton} = 109 \cdot 86b'.$$

$$\text{Weight of hull} = 889 \cdot 54b' \times \cdot 152 \text{ ton} = 135 \cdot 21b'.$$

Also we have

$$\text{I.H.P.} = \frac{14^3 \times \text{mid. sec.}}{516.5} = \frac{2744 \times 43.3b'}{516.5} = 230.3b'.$$

Therefore

$$\text{Nominal H.P.} = \frac{1}{7} \text{I.H.P.} = 32.9b',$$

and

$$\text{Weight of engines and coals} = 65.8b'.$$

Equating the two values of the displacement, as in the former case, we have

$$10.06b'^2 = 135.21b' + 109.86b' + 65.8b' + 700;$$

$$\therefore 10.06b'^2 - 310.87b' = 700$$

and

$$b' = 33.$$

From this value of b' , substituted in former expressions, we get the following dimensions and particulars of the new ship of the 'Bellerophon' type with thick armour, viz.—

Length between the perpendiculars	= 380.8 feet.
Breadth extreme	= 71.16 „
Displacement	= 10958 tons.
Indicated horse-power	= 7599 H.P.
Nominal horse-power	= 1085½ „
Weight of hull	= 4462 tons.
Weight of armour and backing	= 3625 „
Weight of engines and coals	= 2171 „

The foregoing investigation requires a correction which can with advantage be made now. It will be observed that the length of the new ship of the 'Bellerophon' type is less than that of the present 'Minotaur,' and her displacement is not much greater than the actual displacement of the present 'Minotaur' when on sea service. The strength of the present 'Bellerophon,' which is greater than that of the present 'Minotaur,' will therefore suffice for the new ship of the 'Bellerophon' type. But the present strength, and therefore weight of hull, would not suffice for a ship of the dimensions and proportions of the new 'Minotaur,' which is to be nearly 500 feet long, of more than 14000 tons displacement, and heavily armour-plated upon long fine lines. Consequently the strength and weight of hull of the new ship of the 'Minotaur' type must obviously undergo a considerable increase, which it would be unsafe to make less than 10 per cent. upon the present weight of hull.

Making this correction, (1) will become

$$\begin{aligned} \text{Weight of hull} &= 1.1 \times 188.75b \\ &= 207.625b, \end{aligned}$$

and the equation (A) being modified accordingly leads to the quadratic

$$13.2b^2 - 422.37b = 1000,$$

whence

$$b = 34.21,$$

and the dimensions and particulars of the new ship of the 'Minotaur' type become

Length between perpendiculars	=509·7 feet.
Breadth extreme	= 75·5 „
Displacement	=15447 tons.
Indicated horse-power	=7550 H.P.
Nominal horse-power	=1078 „
Weight of hull	=7100 tons.
Weight of armour and backing	=5190 „
Weight of engines and coals	=2157 „

Putting the results into round numbers, writing them down for the two ships side by side and adding the tonnage, we shall have

	New ship of 'Minotaur' type.	New ship of 'Bellerophon' type.
Length	510 feet.	380 feet.
Breadth	75 „	71 „
Tonnage	13770 tons.	8620 tons.
Nominal horse-power	1080 H.P.	1080 H.P.
Indicated horse-power	7560 „	7560 „
Weight of hull	7100 tons.	4460 tons.
Weight of armour and backing	5190 „	3630 „
Weight of engines and coals	2160 „	2160 „
Weight of stores carried	1000 „	700 „
Displacement	15450 tons.	10950 tons.

Here, then, we find that with the same thickness of armour to be carried, and the speed of 14 knots to be attained, the differences resulting from the adoption of the 'Bellerophon' type in preference to the 'Minotaur' type would be a decrease in length of 130 feet, in breadth of 4 feet, in tonnage of 5150 tons, in quantity of armour and backing of 1560 tons, in hull of 2640 tons, and in total weight of ship, or displacement, of 4500 tons. The average cost of the hull of armour-plated ships is about £55 per ton of tonnage, so that the saving in the present ship would amount to £283,250, or considerably more than a quarter of a million sterling.

The engines and coal would cost the same in both cases.

When it is borne in mind that the dimensions of the 'Minotaur' and a tonnage of 6643 tons were attained in that type of ship with almost the earliest armour backing and skin plating, while in the 'Hercules' much greater weights per sq. ft. of surface are carried on a tonnage of 5226 tons (1417 tons less), it will be seen that there is nothing contrary to actual practice in the above estimate of a saving of more than a quarter of a million, large as it undoubtedly is for a single ship. And it must in fairness be added that the ship of the 'Bellerophon' type which would be thus econo-

mical in her first cost, would cost much less than the other for use, for maintenance, and for repairs during the whole period of her existence, and would possess the inestimable advantage of greatly superior handiness in action.

I will now pass from the consideration of existing ships, and take a more abstract case.

Let us presume that we have a ship with prismatic sides vertically, as before, but with curves of sines for horizontal sections.

The weight of hull is to be distributed equally over the sides and bottom.

And let us in the first instance take the proportion of length to breadth to be 7 to 1. It will be found by construction that the length along the curve is to the breadth as 7.1 to 1.

Let b = half breadth extreme.

w = weight per sq. ft. of surface for hull.

$2w'$ = weight per sq. ft. of armour and backing.

d' = depth of armoured side.

$\frac{d'}{4}$ = depth of lower edge of armour below water.

d = draught of water.

$3W$ = weight of equipment (exclusive of engines, boilers, and coal)

s = required speed in knots.

$$\begin{aligned} \text{Surface for weight of hull . . .} &= 4 \times 7.1b(d + \frac{3}{4}d') + 14b^2 \\ &= 28.4b(d + \frac{3}{4}d') + 14b^2. \end{aligned}$$

$$\begin{aligned} \text{Surface for armour and backing} &= 4 \times 7.1b \times d' \\ &= 28.4bd'. \end{aligned}$$

Let us assume in this case that Professor RANKINE'S rule for the calculation of horse-power and speed holds, viz. that the indicated horse-power equals the "augmented surface" multiplied by the cube of the speed and divided by a certain "coefficient of propulsion;" where the "augmented surface" is the immersed surface multiplied by a coefficient of augmentation, which is equal to $1 + 4$ (sine of greatest obliquity of water-line)² + the 4th power of the same sine. The coefficient of propulsion for a clean iron ship of good form is 20000.

In the present case,

$$\text{Angle of maximum obliquity of water-line curve} = \frac{18.0^\circ}{2} \times \frac{1}{7} = 12\frac{6}{7}^\circ.$$

$$\begin{aligned} \text{Therefore coefficient of augmentation} &= 1 + 4 \sin^2 12\frac{6}{7}^\circ + \sin^4 12\frac{6}{7}^\circ \\ &= 1 + 4 \times .2225^2 + .2225^4 \\ &= 1 + 4 \times .0495 + .0024 \\ &= 1.2. \end{aligned}$$

$$\begin{aligned} \text{And augmented surface} &= 14b(2d + b) \times 1.2 \\ &= 16.8b(2d + b), \end{aligned}$$

$$\text{and I.H.P.} = \frac{16.8b(2d + b)s^3}{20000},$$

$$\text{and nom. H.P.} = \frac{2.4b(2d + b)s^3}{20000}.$$

And supposing the engines (including boilers, &c.) to weigh 1 ton per nom. H.P., and the coal to weigh as much as the engines and boilers, we have

$$\text{Weight of engines and coal} = \frac{2.4b(2d+b)s^3}{10000}.$$

Equating the two expressions for the displacement, we shall have

$$\frac{14b^2d}{35} = \{14b^2 + 28.4b(d + \frac{3}{4}d')\}w + 28.4 \cdot b \cdot d' \cdot w' \times 2 + \frac{2.4b(2d+b)s^3}{10000} + 3W. \quad (I)$$

In the new ship let $s=14$ knots, $d=25$ ft., $d'=24$ ft., $W=450$ tons; and while w remains the same as in 'Bellerophon,' say, one-tenth of a ton, let w' be doubled, say, $\frac{6}{28}$ ton; the equation (I) then becomes

$$10b^2 = (14b^2 + 28.4b \times 43) \times .1 + 28.4b \times 24 \times \frac{6}{28} + \frac{2.4b(50+b)2744}{10000} + 1350,$$

or $7.94b^2 = 301b = 1350,$

whence $b = 41.95;$

consequently

Length	$= 14b =$	587.3 feet.
Breadth	$= 2b =$	83.9 ,,
I.H.P.	$=$	8890 H.P.
Weight of hull	$=$	7586.7 tons.
Weight of armour and backing	$=$	6124.7 ,,
Weight of engines and coals	$=$	2541.3 ,,
Weight of equipment	$=$	1350 ,,
Displacement	$=$	<u>17602.7 tons.</u>

The area of the immersed surface of this ship $= 14b^2 + 28.4b \times d.$
 $= 54422$ sq. ft.

and area of immersed mid. sec. $= 83.9 \times 25$
 $= 2097.5$ sq. ft.

Next, let us take the case of a curve-of-sines ship having a proportion of length to breadth of 5 to 1. It will be found by construction that the length along the curve will be to breadth as 5.12 to 1. All the previous notation and values will be retained, except that b' will be substituted for b , and $2W$ for $3W$, the latter change conditioning that this vessel shall carry two-thirds the dead weight of the former vessel.

Surface for weight of hull $= 20.48b'(d + \frac{3}{4}d') + 10b'^2.$

Surface for armour and backing $= 20.48b'd'.$

Taking Professor RANKINE'S rule as before,

Angle of maximum obliquity of water-line curve $= \frac{180^\circ}{2} \times \frac{1}{5} = 18^\circ.$

Therefore coefficient of augmentation $= 1 + 4 \times .0669 + .0049$
 $= 1.273,$

and augmented surface $= 10b'(2d + b') \times 1.273,$

and I.H.P. = $\frac{12.73b'(2d+b')s^3}{20000}$,
 and nominal H.P. = $\frac{1.82b'(2d+b')s^3}{20000}$.
 Weight of engines and coal = $\frac{1.82b'(2d+b')s^3}{10000}$.

Hence we have, equating the two expressions for the displacement, as before,

$$\frac{10b^2d}{35} = \{10b'^2 + 20.48b'(d + \frac{3}{4}d')\}w + 20.48 . b' . d'w' \times 2 + \frac{1.82b'(2d+b')s^3}{10000} + 900 \text{ tons.}$$

Substituting the numerical values, we have $5.64b'^2 - 218.35b' = 900$,
 whence $b' = 42.45$,
 consequently

Length	= $10b' =$	424.5 ft.
Breadth	= $2b' =$	84.9 „
I.H.P.	=	6859.6 H.P.
Weight of hull	=	5540.3 tons.
Weight of armour and backing	=	4470.8 „
Weight of engines and coals . .	=	1959 „
Weight of equipment	=	900 „
Displacement	=	12870.1 tons.

The area of the immersed surface of this ship = $10b'^2 + 20.48b'd$
 = 39753 sq. ft.

and area of immersed midship section = 84.9×25
 = 2122.5 sq. ft.

Collecting the results for the two ships, and writing them down in round numbers side by side, we have

	Ship seven times her breadth.	Ship five times her breadth.
Length	585 ft.	425 ft.
Breadth	84 ft.	85 ft.
Indicated horse-power	8890 H.P.	6860 H.P.
Nominal horse-power	1270 „	980 „
Weight of hull	7586 tons.	5540 tons.
Weight of armour and backing . .	6124 „	4470 „
Weight of engines and coals . . .	2540 „	1960 „
Weight of equipment	1350 „	900 „
Displacement	17600 tons.	12870 tons.

These results are very different in detail from those obtained in the cases based on the actual ships 'Bellerophon' and 'Minotaur,' and on their trials, the shorter ship requiring in this case 2000 indicated horse-power less than the longer ship to drive her at the

given speed of fourteen knots, while the breadth of the short ship slightly exceeds that of the long ship. But these differences in results are not greater than might have been anticipated from the adoption of such different forms of ships, and from the application to them of so different a rule for connecting speed with power.

At the same time the investigation serves to show conclusively, that whether we adopt the theoretical best form of ship, and apply the most recent rule in our calculations, or whether we guide ourselves by the practical trials of existing iron-clad ships, we in either case find that the speed of fourteen knots can be obtained in the short type of ship with surprisingly less size and cost than the long type of ship requires.

It is easy to see how the difference between the horse-powers of the two curve-of-sines ships arises. The immersed surface of the longer vessel exceeds that of the shorter by 14669 sq. ft. At the speed of fourteen knots the frictional resistance per sq. ft. of surface = 1.96 lb., and consequently the excess of this resistance upon the larger ship is $14669 \times 1.96 = 28810$ lbs. This resistance has to be overcome through a distance of fourteen knots per hour; and taking the effective work of one I.H.P. to be equal to 200-knot lbs. per hour (which is the quantity used by Professor RANKINE in his calculation), we obtain

I.H.P. expended in overcoming the excess of frictional resistance upon the larger ship

$$\begin{aligned} &= \frac{28810 \times 14}{200} \\ &= 2016.7 \text{ H.P.}, \end{aligned}$$

which corresponds very nearly to the difference between the engine-power of the two ships. As the immersed midship sections differ so little in the two cases, the difference of horse-power due to this cause would obviously be very slight.

It must be observed that the weight per sq. ft. of hull has been taken the same for both curve-of-sines ships, whereas an increase would be necessary for the larger ship.

I omit this correction as it would lead to a cubic equation; but it would obviously enhance the size and cost of the larger ship, and thus exhibit the advantage of the shorter ship in a still clearer light.